SPIRIT 2.0 Lesson:

Solutions, Solutions, Could it Be?

============Lesson Header =============

Lesson Title: Solutions, Solutions, Could it Be?
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Algebra Topic: Linear Systems
Grade Level: Upper Middle, Secondary
Cartoon Illustration Idea: **A robot with a laser gun and Batman type effects ZIPPPPP**

Outline of Lesson

Content (what is taught):

- Linear Systems
- Experimental Design
- Analysis and inference from data

Context (how it is taught):

- The robot is driven on the floor to reorient the lines created by the laser level light looking for different interactions between the lines
- Pictures are taken to record the position of the lines relative to each other and the grid on the floor.
- The pictures will be analyzed to "arrive" at the number of possible solutions to each system. The grid in the pictures could also be used to calculate the actual solution set for the system.

Activity Description:

In this lesson the number and nature of solutions to a linear system will be explored. Robots equipped with laser levels will "shoot" beams of light making lines. The robots will be driven around on the floor marked with a grid to locate different interactions between lines (this can be done with 2 or more lines). Pictures of each unique situation will be taken from above and saved for analysis.

Standards: (At least one standard each for Math, Science, and Technology - use standards provided)

Math
C2, E1, E2, E3

Science
A1, A2, E1, F5

Technology
C1, C2, D3

Materials List:

Multiple robots, equipped with laser levels
Floor of room marked with a Cartesian grid
Record Sheet
Digital camera
Safety googles (if deemed necessary)

ASKING Questions (Solutions, Solutions, Could it Be?)

Summary: The concept of what is a solution and how many solutions are possible for a linear system will be explored.

Outline:

- Utilize meter sticks to represent lines and explore the possible interactions between lines
- Explore the concept of what a solution of a linear system could be
- Discuss a 3-line system and systems in 3 dimensions

Activity:

The teacher will give students meter sticks to represent lines. Students will be asked what a solution to a linear system looks like and will explore the various possibilities for solutions of a system.

Questions	Answers
What will the solution to a linear system look like?	The solution is where all lines intersect. A point.
What are the different possibilities for lines to interact?	Lines can be parallel, intersecting, and coincide (be exactly the same line).
What would the answer to a system of 3 linear equations look like?	It is still the intersection of ALL lines.
What if you are working in 3 dimensions?	The solution is still the intersection of the lines, but there are a lot more possible no solution cases.

Image Idea: Many different lines represented in a plane and also in 3D.

EXPLORING Concepts (Solutions, Solutions, Could it Be?)

Summary: Robots (equipped with laser levels) will be driven to explore all the possible ways lines can interact. Digital pictures will document different interactions of the lines.

Outline:

- Robots will be driven on the floor marked with a grid. The robots will be equipped with a laser level to help represent the lines. You will need to float some talc in the air to show the lines. Note the room will also need to be darkened
- When unique interactions between the lines are found the placement of the lines will be documented from above by capturing the location of the lines on the grid by digital picture
- The laser levels can be rotated up so that the lines will be in 3 dimensions. This will show that it is much more difficult to find a solution in 3D

Activity: Robots will be driven on the floor to simulate the interactions between lines. It will quickly be come apparent that the case where lines intersect is easy to simulate. This is by far the most common case. The case where lines are parallel or coincide will be harder to simulate with the robots but that is to be expected since there are fewer situations like this. Each unique situation will be documented from above capturing the position of the lines on the grid. Several cases where the lines intersect need to be documented. A third robot can be added and the process can be done again. Finally, the laser levels can be tilted at an upward angle and driven to look for solutions. Students will find that in three dimensions it is much more difficult to locate solutions to the system. All though you cannot document 3D simulations as well, you might try.

Image Idea: Robot modified to carry a laser level
Or a video clip of how you can attach a laser level to a robot?

Instructing Concepts (Solutions, Solutions, Could it Be?)

Filled in by math content writing team

Organizing Learning (Solutions, Solutions, Could it Be?)

Summary: Using the pictures created by the robots, students will look at the relationships of the lines to prove the concepts for consistent/inconsistent and independent/dependant exist in the examples. Data relating to the pictures will be recorded in a chart with the picture included and students will generalize how to tell if there are solutions to a system without graphing it.

Outline:

- Students will look at the data that was recorded previously and explore what is special about each case.
- Utilizing the tools of linear functions, students will write equations for each system using the line and grid.
- Students will analyze what it is about the equations that make them have zero, one or infinite solutions.

Activity:

Using the pictures taken previously, students will look at each case and decide why it has a particular number of solutions. Next, the equations of the lines will be calculated using the grid as a reference. Notations about why each case has a certain number of solutions and the equations calculated will be recorded on a data sheet including the picture. Students will then explore the relationship between the number of solutions each system has and the equations (zero solutions – parallel, infinite solutions – equations are multiples of one another, and intersecting – not one of the other cases).

Three-dimensional cases that were recorded can be discussed even though the equation cannot be found. The fact that the parallel lines and skew lines both provide no solution for the system can be inferred from the data.

Linear Function Picture #	Line 1 Equation	Line 2 Equation	Line 3 Equation	# and Type of Solution	Similarities of Lines

Understanding Learning (Solutions, Solutions, Could it Be?)

Summary: Students will write an essay on the number and type of solution to a linear system. Another possible assessment would be to present various systems to student and have them algebraically find the number and nature of solutions.

Outline:

- Formative assessment of linear systems
- Summative assessment of linear systems

Activity:

Formative Assessment

As students are engaged in the lesson ask these or similar questions:

1) Are students able to apply explain what a solution to a linear system is and why?
2) Can students explain how many solutions are present in a given system?
3) Can students tell how many solutions there are to a system without graphing it?

Summative Assessment

Students will be asked to write a formal lab write-up with the experimental procedure, the data, and the relationships calculated of 2 linear systems photographed. They will then be given a system of linear equations and asked to determine if there are solutions and how many are present.

Students will answer the following writing prompt:

1) Explain how the graph of a system relates to the number of solutions.
2) Explain how many answers can there be to a linear system and how you determine this information without graphing.

Students could answer these quiz questions as follows:

1) How many solutions does the following system have:

$$\begin{cases} 3x + 5y = 7 \\ 5x - 3y = 18 \end{cases}$$

2) Write a linear system in 2 dimensions that has no solutions.